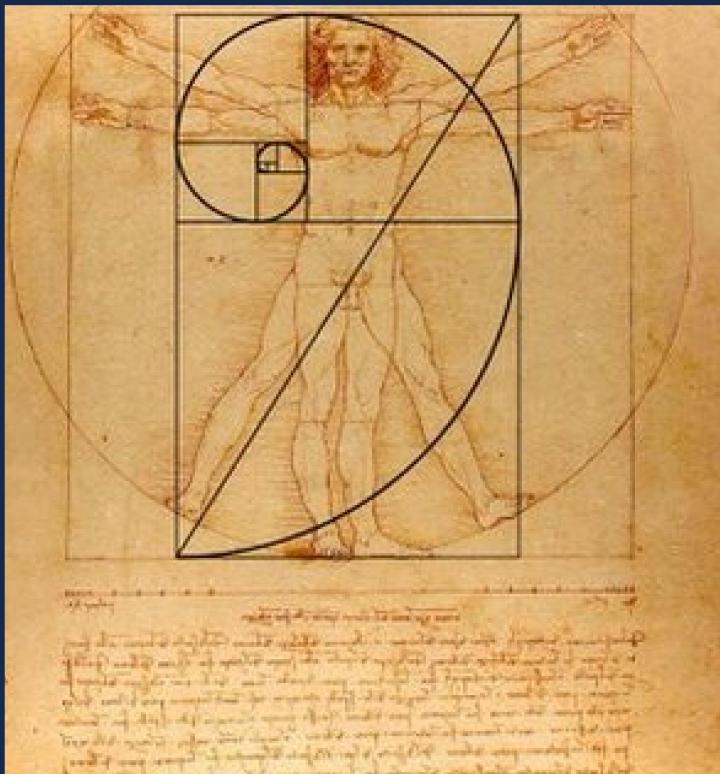


# THE PHI CODE

Nature, Architecture, Engineering



RAMÓN FELIPE BORGES, Ph.D

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# INTRODUCTION

This book contains papers written and presented by the author at international conferences on Heritage Architecture.

The conferences took place in various venues with the participation of specialists on cultural heritage well known all over the world.

The importance of historic buildings for the cultural identity of the region or nation is well recognized and appreciated requiring looking beyond borders to benefit from the experience of others.

In this volume, the author analyses topics related to structural, historical, and the architecture of culturally meaningful buildings.

Chapter 1, "The phi code in nature, architecture and engineering"; is devoted to studying the golden ratio,  $\phi$  (Phi), as it appears systematically coded (the phi code) in nature, architecture, and engineering. The phi code helps explain the growth patterns in nature, the aesthetics of design in architecture, and the behavior of structural elements used in engineering.

The phi code defines the morphogenesis (development of forms) in nature and the aesthetics of forms in architecture and its structural parts.

Chapter 2 , "The Golden Mean and the Dome of San José Cathedral"; a structural interpretation"; relates to a study conducted by the author to understand better the architecture, history, and structural characteristics of San Jose Cathedral, San Juan, Puerto Rico.

It discusses the structural action of the dome that covers the Rosary Chapel of the cathedral and the creation of numerical models accounting for the material behavior of the masonry roof structure. Furthermore, the golden mean is employed to interpret the structural behavior of the hemispherical dome, particularly regarding the stress resultants acting in the plane of the shell and the horizontal thrust developed at the base of the dome covering the Rosary Chapel.

Chapter 3 , "The concept of proportion in heritage architecture: a study of form, order, and harmony"; examines how different concepts of proportion used by Western civilization were employed to create "form, order, and harmony" in architectural heritage structures. The study covers the period from Antiquity to the Middle Ages and how a particular system of proportion was applied to extant monuments constructed by the ancient civilizations of Egypt and Greece.

Chapter 4 , "Evolution of architectural forms of historic buildings"; examines the evolution of architectural forms of historic masonry buildings up to the middle ages. Consideration is given to those building forms or structures that contributed most to the stability, aesthetics, and topology of the monument.

As presented in this book, all these developments brought about structural concepts and novel methods of construction that revolutionized the architecture of historic buildings.

In particular, how the "golden ratio" (the phi-code) defined the morphogenesis (development of forms) and aesthetics of forms in architecture and its structural parts.

The thesis proposed in the book emphasizes the important role played by the "phi-code" in the development of architectural forms and in defining structural aspects of historic buildings.

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## *Chapter 1.*

# **The phi code in nature, architecture and engineering ( Stress Analysis )**

## **Abstract**

This paper examines how the golden ratio,  $\Phi$  (Phi), appears systematically coded (the phi code) in nature, architecture and engineering. The phi code helps to explain the growth patterns in nature, the aesthetics of design in architecture and the behavior of structural elements used in engineering. The phi code defines the morphogenesis (development of forms) in nature as well as the aesthetics of forms in architecture and its structural parts.

*Keywords:* *golden ratio, golden mean, golden section, divine proportion, geometric proportion, biomimetics, design and nature, architecture.*

## **1 Introduction**

The golden ratio, phi, is coded symbolically manifesting itself in geometric patterns and forms and in mathematical expressions defining the behavior of natural and physical phenomena. As discussed in this paper, certain geometries describe growth patterns in nature, whereas other geometric shapes are employed in art and architecture in order to establish a sense of harmony and proportion in design. In engineering, the phi code appears in mathematical expressions related to structural mechanics particularly in the area of stress analysis of structural elements. The phi code identifies aspects of nature, architecture and engineering in which the golden ratio plays a determinant role.

The phi code establishes rules of behavior. In nature it is growth behavior. In architecture it is aesthetic behavior and in engineering it is structural behavior. In nature, the golden ratio defines patterns of growth in the form of spiral shapes and pentagonal geometries. In architecture, some styles and forms are created by employing different systems of proportion, most of which following golden ratio

proportions in the form of geometric shapes [1]. In engineering, the golden ratio also plays a significant role in the area of stress analysis of basic structural elements (columns, beams, domes) as predicted by elastic theory. In the context of this paper, it is not feasible to discuss all aspects of the phi code in detail. This would entail a lengthy digression into the history of “phi”. In consequence, this article discusses various themes of the phi code, as they are relevant to nature, architecture, and engineering (structural mechanics). The present work examines in a condensed form, how the phi code is embedded in the growth pattern of shells and pinecones. It also discusses how the golden ratio was employed by the Greeks in the Parthenon. And finally, the paper describes how this ratio defines the structural behavior of simple beams as characterized by the principal stresses. But first, let us discuss some of the properties of the golden ratio to better understand the theme of the phi code.

## 2 The golden ratio $\Phi$ (phi)

The golden ratio is a proportion found in nature. The Ancient Greeks applied it to the design of their temples [2]. In graphical terms, it can be described as “*the whole is to its larger part, what the larger part is to the smaller*”. It is also known as the golden mean, the golden section, or the divine proportion. As shown in Figure 1, it is a way to divide a line in such a manner as to create an ideal relationship between the parts.

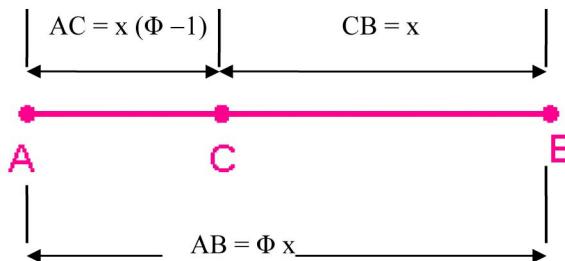


Figure 1: Golden ratio of a line.

From the figure,  $AB/CB = CB/AC$  or,

$$\frac{\Phi x}{x} = \frac{x}{x(\Phi - 1)}, \text{ that is:} \quad (1)$$

$$\frac{\Phi}{1} = \frac{1}{\Phi - 1}$$

$$\Phi^2 - \Phi - 1 = 0 \quad (\text{solving for } \Phi \text{ we obtain } \Phi = \frac{1 + \sqrt{5}}{2} = 1.61803) \quad (1a)$$

The qualities of the golden ratio  $\Phi$  (phi) were central to the numerological philosophy of Plato and Pythagoras. The mathematician Filius Bonacci (Fibonacci) wrote a treatise on the number series related to the golden ratio. 0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597. If we take the ratio of

two successive numbers in Fibonacci's series, we find:  $1/1=1$ ;  $2/1=2$ ;  $3/2=1.5$ ;  $5/3=1.666$ ;  $8/5=1.6$ ;  $13/8=1.625$ ;  $21/13=1.61638$ ;  $34/21=1.61904$ ;  $55/34=1.6176\dots$ ;  $610/377=1.61803\dots$ ;  $1597/987=1.6180$ . The ratio converges to a value very close to the golden ratio ( $\Phi$ ) just as for the "line" above! The Greeks studied phi closely through their mathematics and used it in their architecture. The Parthenon at Athens is a classic example of the use of the golden ratio, as employed geometrically in a rectangular shape. This shape, known as the golden rectangle, is shown in Figure 2.

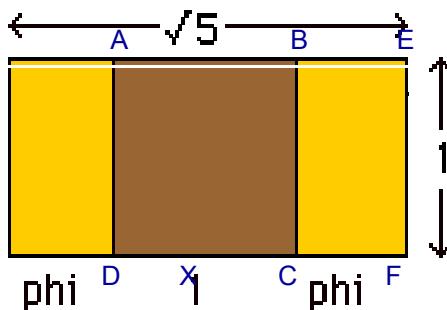


Figure 2: The golden rectangle.

The golden rectangle is produced mathematically when the side DC of a square ABCD is bisected at X and an arc with radius XB is swung on to DC at F. The resultant rectangles AEFD and BEFC are both golden rectangles whose sides conform to the golden ratio, thus:  $DF/AD = BC/CF = 1.618$ .

Golden rectangles can be approximated by using Fibonacci series to construct "Fibonacci rectangles". Fibonacci rectangles are those built to the proportions of consecutive terms in the Fibonacci series  $\text{Fib}(n) = 1, 1, 2, 3, 5, 8, 13, 21, 34, \dots, 55, 89, 144, \dots$ . Because of the nature of the series, any  $\text{Fib}(n+1)$ : $\text{Fib}(n)$  rectangle can be divided into all the previous Fibonacci rectangles. For example, Figure 3 shows a 34:21 rectangle (the numbers inside the squares show the length of the sides) within which is contained the 21:13, 13:8, 8:5, 5:3, 3:2, 2:1 and 1:1 rectangles. Thus, each of the rectangles is divided into squares of length of side conforming to the Fibonacci series. As  $n$  tends to infinity, the proportion of the rectangle will tend to phi. This is a 'very good' approximation of the golden rectangle discussed previously.

Also of interest is the Fibonacci spiral, shown in Figure 4 constructed using the diagonal arcs (arcs of demi-semi-circles) of radius  $\text{Fib}(n)$  for all natural  $n$  up to a given point (depending on the required size/complexity of the spiral). The diagram shows a spiral constructed for all  $n$  up to  $n=21$ , following Figure 3. Let us now turn our attention to the Fibonacci spirals and see how they exemplify the appearance of the phi code in nature.

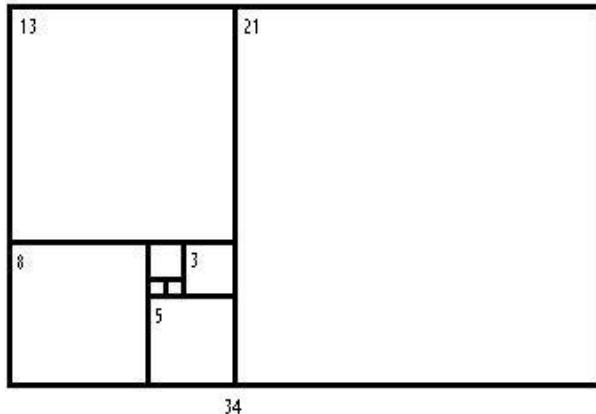


Figure 3: Fibonacci numbers in golden rectangle.

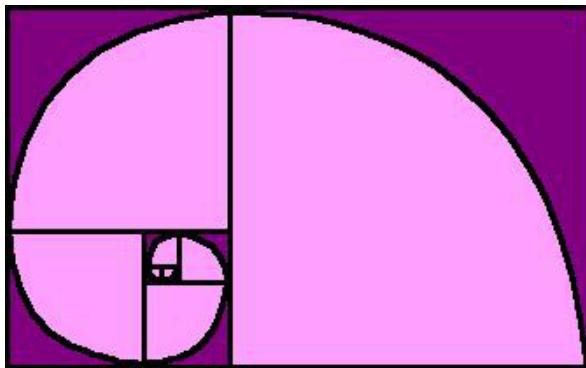
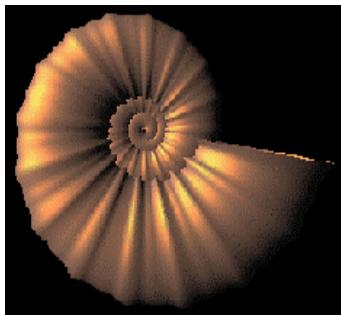


Figure 4: Logarithmic spiral of Fibonacci series: typical of shell growth.

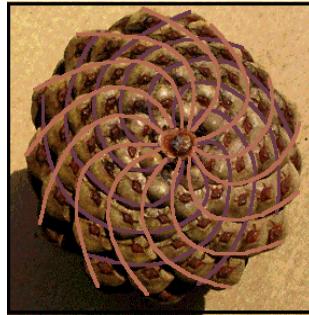
### 3 Phi code in nature

Fibonacci spirals occur widely in nature as for instance, the growth pattern of shells and pinecones shown in Figure 5 (a) and (b) respectively.

In each growth phase characterized by a spiral, the new spiral is very close to the proportion of the golden ratio (a golden rectangle to a square) larger than the previous one. The contour spiral shape of shells, illustrated in Figure 5 (a), reveal a cumulative pattern of growth. The growth patterns of shells are logarithmic spirals which are close to golden ratio proportions, as illustrated in Figure 4. The growth patterns of the nautilus and other shells are never in the exact golden ratio proportions. The nautilus shell is a logarithmic spiral. Such a shape arises because a growing animal has the same proportions as it grows and the spiral fits the requirement to protect this shape as it gets larger.



(a) Nautilus shell



(b) Pinecone spiral

Figure 5: Spirals in nature.

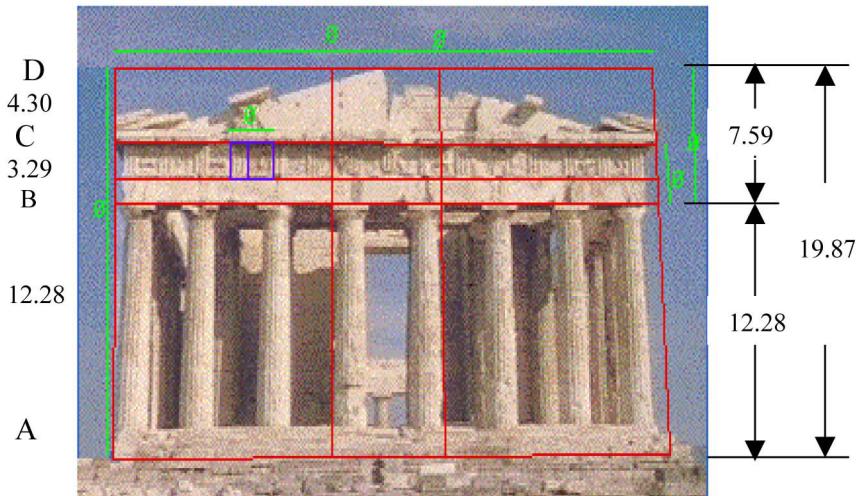


Figure 6: The Parthenon, Athens.

The spiral growth patterns of the pinecone each grow along two intersecting spirals, which move in opposite directions. Upon examining the pinecone seed spirals, shown in Figure 5 (b), eight (8) of the spirals move in a clockwise direction and thirteen (13) in a counterclockwise direction, closely approximating golden ratio proportions.

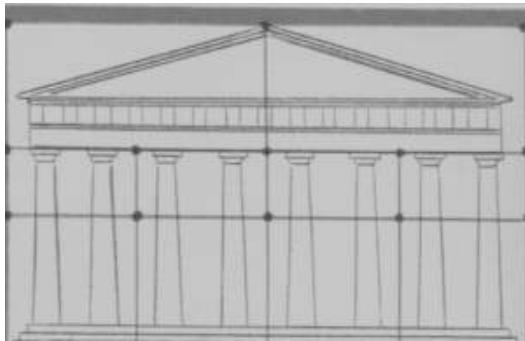
That is  $13/8 = 1.625$ . The numbers 8 and 13 as found in the pinecone spiral are adjacent pairs in the mathematical sequence of the Fibonacci series as shown previously. The ratio of adjacent numbers in the sequence progressively approaches the golden ratio of 1:1.618. The phi code in nature is also found in the proportions of the human body. As an example, consider the ratio of a person's total height to the distance from the navel to the foot. These measurements were made in a number of people and the ratio turned out to be

very close to the golden ratio! [3]. After all, “Man is the measure of all things,” according to Protagoras, the Greek philosopher of the fifth century B.C.

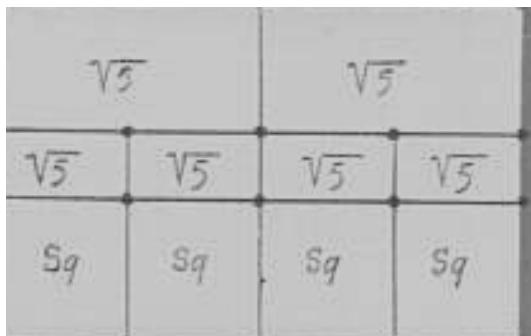
## 4 Phi code in architecture

The Parthenon is an interesting example of a mathematical approach to art. Once its ruined triangular pediment is restored, the ancient temple fits almost precisely into a golden rectangle, Figure 6. The height (in meters) of the building follows golden ratio proportions [4].

That is:  $AD/AB = AB/BD \dots 19.87 / 12.28 = 12.28 / 7.59 \approx 1.618 = \Phi$ . In addition, as shown in Figure 7, the facade of the Parthenon was designed around the proportions of two large and four small golden ratio, or  $\sqrt{5}$ , rectangles (refer to Figures 7a, 7b), placed above four squares [4]. These proportions are in agreement with the golden rectangle, shown in Figure 2.



(a)



(b)

Figure 7: The Parthenon and the golden ratio.

## 5 Phi code in engineering (Stress Analysis)

Let us consider the beam shown in Figure 8 (a). A stress analysis of the beam results in the plane stress condition shown in Figure 8 (b).

The Normal Stresses  $\sigma_x$ ,  $\sigma_y$ , and Shear Stresses  $\tau_{xy}$ ,  $\tau_{yx}$  are acting as shown in the figure for a given x-y coordinate system. The Principal Stresses  $\sigma_1, \sigma_2$  (critical in stress analysis) are also illustrated in Figure 8 (b).

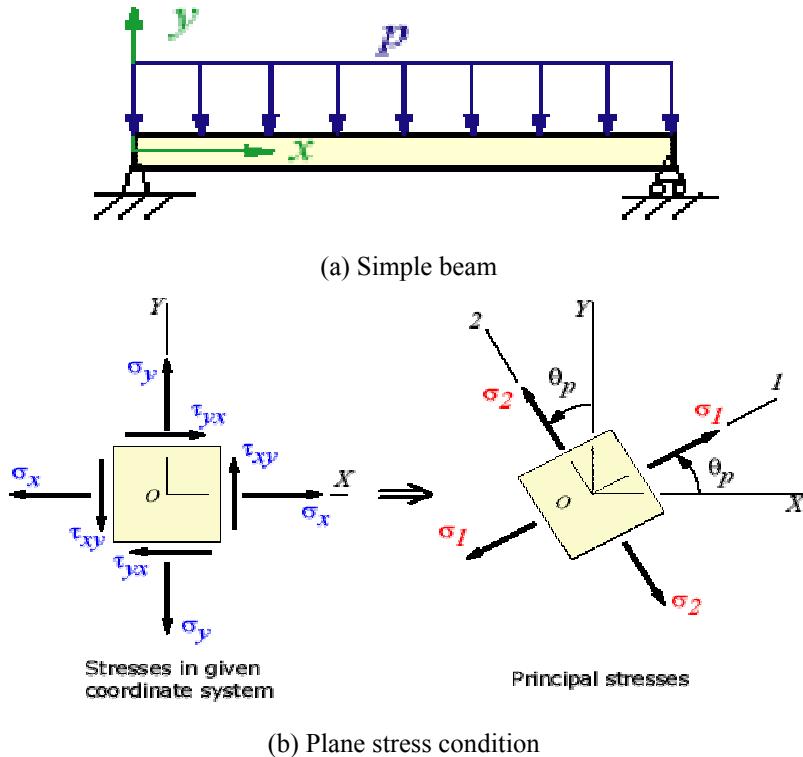


Figure 8: Stress analysis of a simple beam.

The Principal Stresses  $\sigma_1$  and  $\sigma_2$  are given by:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (2)$$

And the Shear Stresses by:

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (3)$$

Normally, beam behavior entails flexural and shearing actions in which  $\sigma_y$  is negligible when compared with the other stresses acting on the beam. That is  $\sigma_y = 0$  for all practical purposes. In addition, for the particular condition in which  $\sigma_x = \tau_{xy}$ , (and  $\sigma_y = 0$  as indicated earlier) we obtain the following relationships for the principal stresses:

$$\sigma_{1,2} = \sigma_x [ \frac{1 \pm \sqrt{5}}{2} ]; \text{ and} \quad (4)$$

$$\sigma_1 = \sigma_x [ 1.61803 ] = \sigma_x [\Phi] \text{ tension} \quad (4a)$$

$$\sigma_2 = \sigma_x [ -0.61803 ] = \sigma_x [ -1/\Phi ] \text{ compression} \quad (4b)$$

The Maximum Shear Stress is given by:

$$\tau_{\max} = \sigma_x [ \frac{\sqrt{5}}{2} ], \text{ Where } \sqrt{5} = \frac{1 + \Phi^2}{\Phi} \quad (5)$$

Indeed, the golden ratio  $\Phi$  definitely is a defining parameter in the stress analysis of beams. See also reference [5] for a discussion on “the golden ratio and the stress analysis of domes”.

## 6 Conclusion

We have seen how the phi code identifies growing patterns in nature in the form of Fibonacci spiral shapes. How the phi code identifies architectural designs in geometrical golden ratio proportions.

And finally, it was shown how the phi code also identifies aspects of engineering in the stress analysis of simple beams. In all of these areas, the golden ratio, phi, plays a significant and determinant role. The geometric forms and mathematical expressions already discussed (Fibonacci spirals,  $\Phi$ -rectangles and principal stresses) represent, on the whole, rules of behavior associated with the specific category (nature, architecture, engineering) under consideration. As discussed in this paper, in nature it is growth behavior, in architecture it is aesthetic behavior and in engineering it is structural behavior.

In conclusion, “the golden ratio” appears to be the principal invariant in patterns of growth described by nature as well as among the various systems of proportion employed in architecture. It is also a significant parameter

encountered in the area of stress analysis when considering the principal stresses acting on a beam as predicted by elastic theory. The phi code, in the form of the golden ratio, is clearly a pervasive theme in nature, architecture, and structural mechanics. The golden ratio may very well be a definitive characteristic of “design” in nature, architecture, and engineering (structural mechanics) as well as an important element of aesthetic expression in all of these areas. Our fascination with the golden ratio and all it represents will continue to be of interest for scientific speculation and artistic expression. We need only to search our environment and hope to discover these  $\Phi$ -geometries and  $\Phi$ -relationships that are encoded in that universe in order to decipher the phi code.

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## *Chapter 2.*

# **The Golden Mean and the Dome of San Jose Cathedral: a structural interpretation**

## **Abstract**

A study conducted by the author is aimed at deriving a better understanding of the architecture, history, and structural characteristics of San Jose Cathedral, San Juan, Puerto Rico. This paper discusses the structural action of the dome that covers the Rosary Chapel of the cathedral and the creation of numerical models accounting for the material behavior of the masonry roof-structure. Furthermore, the golden mean is employed to interpret the structural behavior of the hemispherical dome, particularly in regard to the stress resultants acting in the plane of the shell and the horizontal thrust developed at the base of the dome.

## **1. Introduction**

The San Jose Cathedral [1], is one of the first works of architecture built in the New World by the Spaniards in the 16<sup>th</sup> century. Elements of the Gothic style are found in the construction of the rib-vaults above the Main Chapel and Transept of the church and barrel vaults, of the Italian Renaissance period, cover the Central Nave of this most exceptional example of religious architecture in the Americas.

The masonry dome covering the Rosary Chapel became an integral structural and architectural element of the building. The structural behavior of the rounded vault is studied considering the overall shape of the vault (hemispherical dome), rather than its construction, since it is its geometry (the golden mean) that ultimately controls structural action.

A preliminary assessment of the structural performance of the dome was conducted using membrane shell theory to predict its structural behavior. This approach allows us to develop a feeling for the behavior of the vault and to obtain the primary forces acting in the structure.

## 2. The Hemispherical Dome

### 2.1 General Considerations

The dome covering the Rosary Chapel, Figure 1, is as old as the Main Chapel of the church. The vault, build with stone blocks, rests on four Roman arches built on a square plan measuring 26 ft. (7.93m) on each side. The dome proper, spans a distance of 24 ft. (7.32m). The change from circular to square is achieved with pendentives, a Byzantine invention.

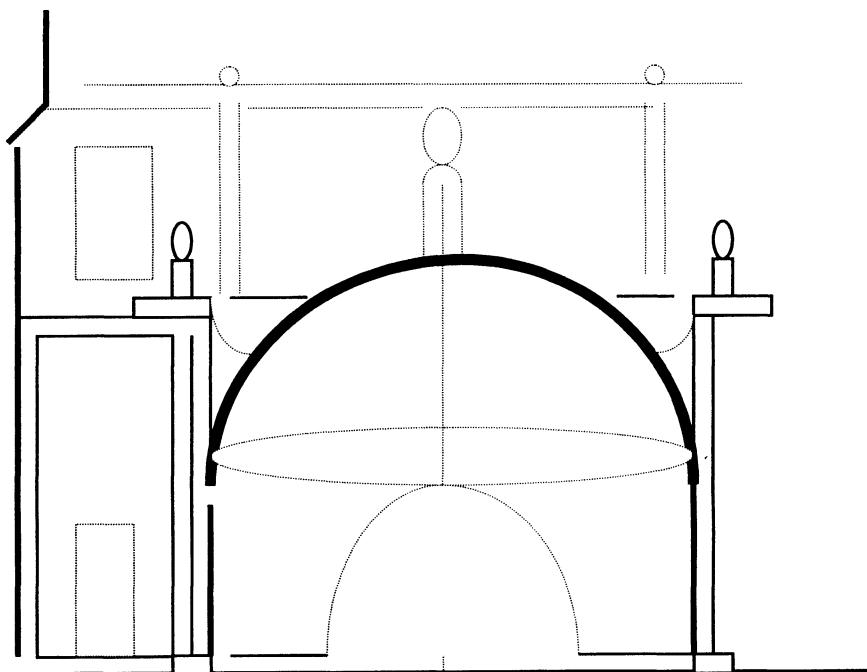


Figure 1. Dome of Rosary Chapel

In this study, the dome is conceived as a hemispherical shell structure which can be idealized as a curved surface in conformance with membrane shell theory. Loads acting on such a surface must be resisted by forces within the surface. In membrane theory it is assumed that the surface has no stiffness against bending, so that the forces in the shell are purely tensile or compressive (tensile forces are inadmissible for masonry, a crucial departure from conventional theory).

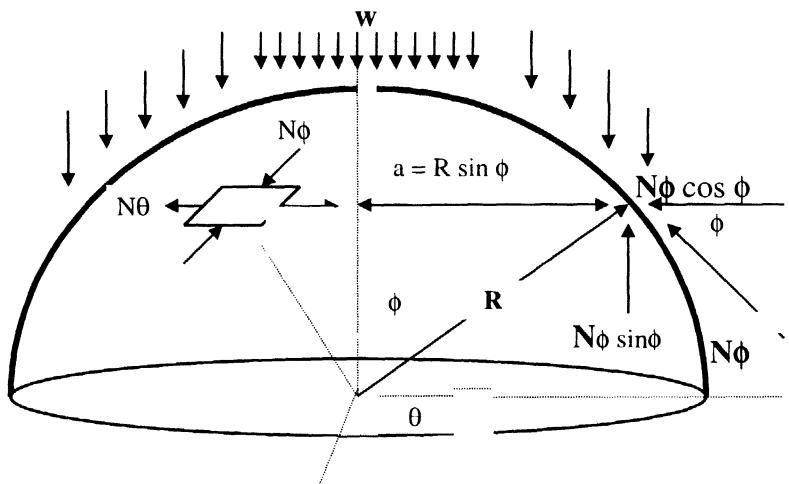


Figure 2. Forces in spherical shells

Figure 2, illustrates the forces acting in spherical shells through its middle surface. This idealized skeletal structure was used as a model to employ membrane shell theory and to predict the primary vault forces. In its simplified form, membrane theory for shells assumes that the basic resistance of the shell to load is by in-plane tension and compression.

## 2.2 Stress Resultants

The behavior of the masonry vault can be examined by assuming that masonry has no tensile strength and that it possess an unlimited compressive strength in light of the low compressive stresses developed. Finally, that sliding failure does not occur although slippage of individual stones may sometimes happen, the structure retains its stability remarkably well [2]. Based upon these assumptions, the concept of stability and hence, geometrical considerations takes precedence and in fact, replaces the more conventional idea of strength as a factor of design in masonry construction. That is, the stability of the structure is assured primarily by its shape and not by the strength of the masonry.

As will be shown later, the golden mean accurately predicts the structural behavior of the dome by establishing the position of the neutral plane thus defining the internal resistance of the structure as well as controlling the horizontal thrust at the base of the dome. As indicated earlier, the skeletal structure of Figure 2, modeling the dome was used to predict the primary forces employing membrane theory.

Referring to Figure 2, the meridional forces (stress resultants),  $N\phi$ , acting on an element of a shell structure, considering  $W$  to be the weight per unit area of shell surface, are of the order:

$$N\phi = \frac{Rw}{1 + \cos \phi} \quad (1)$$

whereas the circumferential or hoop forces (stress resultants),  $N\theta$ , are given by;

$$N\theta = \frac{1}{Rw} (- \frac{1}{1 + \cos \phi} + \cos \phi) \quad (2)$$

### 2.3 Forces on the Dome

Of primary importance in the analysis of shell structures is the existence of these two sets of internal forces in the middle surface of the shell that act in perpendicular directions. A general observation of the vault of San Jose Cathedral was conducted indicating that a preliminary analysis could be made of the structural behavior of the masonry dome using membrane theory. As indicated earlier, the dome over the Rosary Chapel spans a distance of 24 ft. (7.32m). The average thickness,  $t$ , of the shell is 8 in. (203 mm), and the unit weight,  $\rho$ , of the material is 135 lb/ft<sup>3</sup>. (21.23 kN/m<sup>3</sup>). This corresponds to a load,  $w$ , of 90 lb/ft<sup>2</sup>. (4.31 kN/m<sup>2</sup>) of the vault. The radius,  $R$ , of the spherical dome is 12 ft. (3.66 m).

If we consider the maximum stress resultants developed at the springing of the dome and letting  $\phi = 90^\circ$ , then from Equations (1) and (2) we obtain the forces acting in the shell to be equal to:

$N\phi = -N\theta = Rw = (12) \times (90) = 1080 \text{ lb/ft.}$ , the negative sign indicates a tensile force.

Therefore a compressive stress (and tensile stress) of  $\sigma\phi = N\phi / t = 1620 \text{ lb/ft}^2$  or  $11.25 \text{ lb/in}^2$  ( $0.078 \text{ N/mm}^2$ ) acts at the base of the dome. Compared with the typical crushing stress of  $6000 \text{ lb/in}^2$  ( $41.38 \text{ N/mm}^2$ ) for sandstone

masonry and a tensile “strength” equal to about 10% of the crushing stress 600 lb/in<sup>2</sup> (4 N/mm<sup>2</sup>) for masonry (tensile bond strengths for mortar are in the order of about 0.5 N/mm<sup>2</sup> to 1.5 N/mm<sup>2</sup>). A more than adequate level of safety for the shell.

Thus, the stresses in the dome at this point ( $\phi=90^\circ$ ) are extremely low if we compare them with the strength of the masonry. This is characteristic of most shell structures carrying their own weight.

As indicated earlier, the dome of San Jose Cathedral is supported by four arches built on a square plan. The transition from the dome circular horizontal section to the square plan of the chapel is achieved by the use of pendentives leading from the angle of the arches to the base of the dome. The masonry dome thrust against these supports, and must be buttressed by those supports. If we consider masonry as a no-tension material, the tensile forces toward the base of the dome are inadmissible for the no-tension masonry structure, and a different analysis (as that provided by membrane theory) must be used to explain satisfactorily the action of the masonry dome. The stress resultants of Equations (1) and (2) are determined from the analysis of a truly hemispherical shell of small thickness. The actual thickness of the real dome allows for the thrust line of action not to be restrained to follow the center line of the shell. Therefore, the inclined thrust line at the base of the dome will produce a lateral thrust action against the dome supports. These abutments may give way slightly unless the base of the dome is encircled by a tie to resist tension.

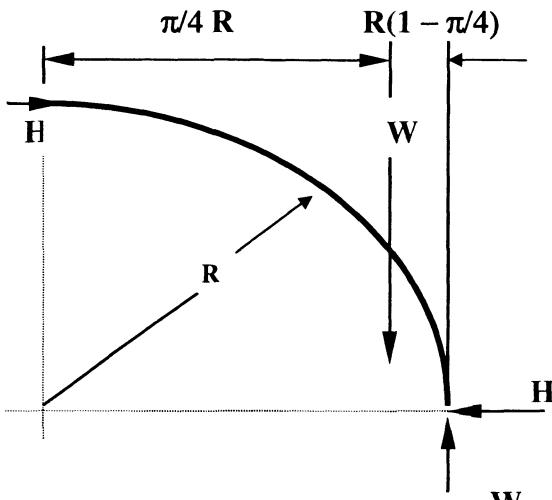


Figure 3. Forces on segment of dome

Based upon these assumptions and considering the forces acting on a segment of a dome, as shown in Figure 3, then considering static equilibrium the total horizontal force  $\mathbf{H}$  uniformly distributed around the base is given by:

$$\mathbf{H} = (1 - \pi/4) \mathbf{W} = 0.215 \mathbf{W}, \text{ where } \mathbf{W} \text{ is the total weight of the dome. (3)}$$

Thus, for the hemispherical dome spanning 24ft. ( 7.32m), with a surface area of  $A = 2\pi R^2$ , weighting  $w = 90 \text{ lb/ft.2}$ , the total weight,  $W$ , is:

$\mathbf{W} = \mathbf{w} \times \mathbf{A} = 90 \times 2\pi (12)^2 = 81\,430 \text{ lbs. (362.2 kN) }$ . Hence, the total horizontal thrust would be  $\mathbf{H} = 17507 \text{ lbs. ( 77.87 kN)}$  this corresponds to a uniformly distributed force around the base of

$\mathbf{p} = 232 \text{ lb/ft (3.4 kN/m)}$ . If this force were to be resisted by a tension ring at the base of the dome, the total tie force,  $\mathbf{T}$ , would then be equal to:

$$\mathbf{T} = \mathbf{p} \times \mathbf{R} = 232 \times 12 = 2784 \text{ lb.(12.38 kN)}. \quad (4)$$

These figures are of a very low order of magnitude (the dome of St Peter's Rome yields  $\mathbf{p} \approx 300 \text{ kN/m}$ ) for the horizontal thrust, which in the absence of proper ring ties, is contained by the masonry abutments to the dome. These abutments provide the necessary support and stability for the whole structure.

In the next section, the golden mean is employed to predict the structural behavior of the dome in light of the results obtained previously when using membrane theory.

### 3. The Golden Mean

#### 3.1 The Golden Ratio

The Golden Mean is a proportion found in nature. It was developed by the Ancient Greeks who applied it to designing their temples. It is described as "the *whole* is to its *larger section*, what the larger section is to the *smaller*." It is also known as the Golden Section, the Golden Ratio, or the Divine Proportion. As shown in Figure 4, it is a way to **divide a line** in such a way as to create an ideal relationship between the parts.

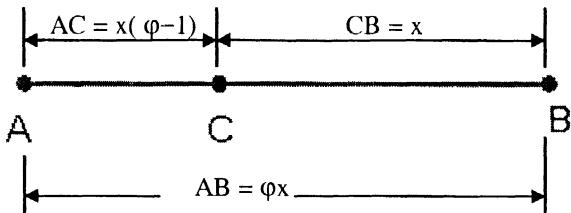


Figure 4. Golden Ratio of a line

From the figure,  $AB/CB = CB/AC$  or,

$$\frac{\varphi x}{x} = \frac{x}{x(\varphi - 1)}, \text{ that is:}$$

$$\varphi^2 - \varphi - 1 = 0 \quad (\text{solving for } \varphi \text{ we obtain } \varphi = \frac{1 + \sqrt{5}}{2} = 1.61803). \quad (5)$$

Throughout history [3], Phi has been observed to evoke emotion or aesthetic feelings within us. The ancient Egyptians used it in the construction of the great pyramids and in the design of hieroglyphs found on tomb walls. At another time, the ancients of Mexico embraced Phi while building the Sun Pyramid at Teotihuacan. The Greeks studied Phi closely through their mathematics and used it in their architecture. The Parthenon at Athens is a classic example of the use of the **Golden Rectangle**.

The Golden Rectangle, shown in Figure 5, is produced mathematically when the side DC of a square ABCD is bisected at X and an arc with radius XB is swung on to DC at F. The resultant rectangles AEFD and BEFC are both golden rectangles whose sides conform to the golden mean, having the ratio 1: 1.618

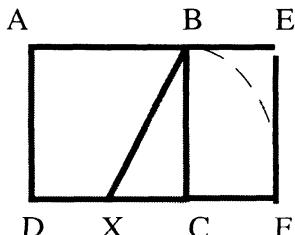


Figure 5. Golden Rectangle

The golden mean has been utilized by numerous artists and scientists since (and probably before) the construction of the Great Pyramid (c.2500 BC). As scholars and artists of past eras discovered (i.e.: Plato, Pythagoras, Leonardo da Vinci), the *intentional* use of the golden mean in art of various forms expands our sense of beauty, balance, and harmony to optimal effect.

### 3.2 The Golden Ratio and the Structural Behavior of the Dome

Reflecting upon what may be the simplest and perfect of forms, the sphere is an ultimate expression of unity, completeness, and integrity. By utilizing geometric proportions leading to the golden ratio (i.e.,  $\Phi = 1.618\ldots$ ), it is possible to predict the **structural behavior of the dome** (an *unintentional* occurrence)! by considering the overall spherical shape of the masonry structure, rather than its construction, since it is its geometry (in accordance with the golden ratio) that ultimately controls structural action.

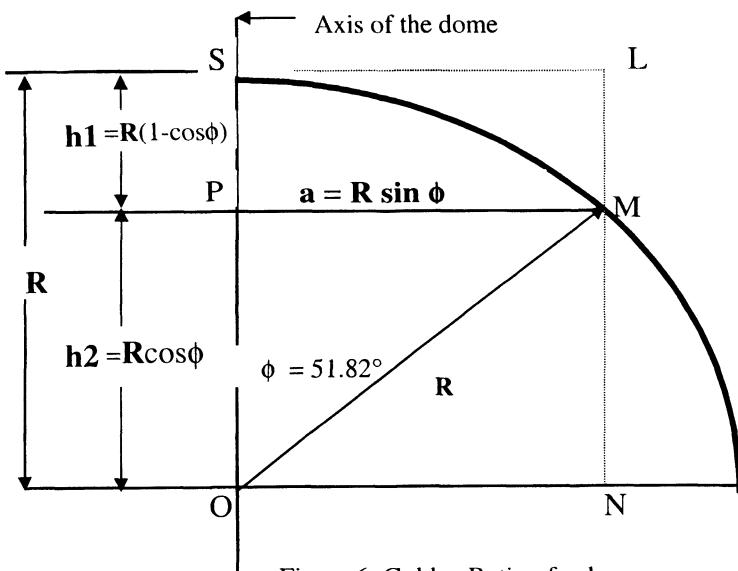


Figure 6. Golden Ratio of a dome

Referring to Figure 6, and considering the stress resultants acting on the spherical dome, it is seen from Equation (2) by letting  $N\theta = 0$  (thus locating the neutral plane) that the resulting equation:

$$\cos 2\phi + \cos \phi = 1 \dots\dots\dots \text{has the solution } \phi = 51.82^\circ. \quad (6)$$

At this co-latitude..the meridional stress,  $N\phi$ , from Equation (1) gives:

$N\phi = R_w / 1 + \cos(51.82^\circ) = R_w / 1 + 0.618 = R_w / 1.618$  or  $R_w/\phi$ , where  $\phi = 1.618$ ...the golden ratio.

This angle  $\phi = 51.82^\circ$ , which can be termed "the golden angle," defines a point, M, on the meridian which is at a distance  $a = R \sin\phi$  from the vertical axis of the dome. This distance  $a$  defines in turn, the vertical distances  $h_2 = R \cos\phi$  and  $h_1 = R(1 - \cos\phi)$ , effectively dividing the vertical axis of the dome (that is the radius  $R = h_2 + h_1$ ) into golden sections,

$$R/h_2 = h_2/h_1 = 1.618 \dots, \text{ again the golden ratio.}$$

Evidently the ratio of the areas of the resulting rectangles (golden rectangles), also yield the golden ratio, thus:

$$\frac{\text{OSLN}}{\text{OPMN}} = \frac{\text{OPMN}}{\text{PSLM}} \dots \text{ or } \dots \frac{R a}{h_2 a} = \frac{h_2 a}{h_1 a}$$

Furthermore, if we consider Equation (6)  $\cos^2\phi + \cos\phi = 1$ , resulting by letting the hoop stress  $N\theta$  equal to zero (a structural consideration) and Equation (5)  $\phi^2 - \phi = 1$ , obtained when considering the golden mean as applied to a line or a rectangle (a geometric consideration) then, it is evident that for both quadratic equations the solution is of the form:

$$\phi = \frac{1 + \sqrt{5}}{2} = 1.61803\dots$$

Further reference to Figure 3 and Figure 6, reveals that the line of action of the total weight,  $W$ , of a segment of a dome, passes through a point, M, on the meridian defined by the "golden angle"  $\phi = 51.82^\circ$ .

That is  $a = R \sin\phi \equiv \pi/4 R$  or  $\pi/4 \equiv \sin(51.82^\circ) = 0.786$ . Therefore, the "golden angle"  $\phi = 51.82^\circ$  controls the magnitude of the horizontal thrust,  $H$ , at the base of the dome, by way of the moment arm of  $W$ . Thus taking moments about the base of the dome, we have:

$$HR = R(1 - \pi/4) W \text{ or } H = \{(1 - \sin(51.82^\circ)\} W \dots \dots \text{ As given by Equation (3)}$$

## **4. Comments**

The proper understanding of the behavior of a masonry dome is to be found in the correct interpretation of geometry and, as we have seen, its correlation with the "golden ratio,"  $\Phi = 1.61803\dots$  A proportion that actually controls the mechanism of structural behavior of a spherical shell under self-weight loading conditions.

A significant aspect of this preliminary study that still needs to be ascertained is to determine the structural action of the spherical shell at the base of the dome and its interaction with the supports as predicted by a more accurate mathematical model representing the "actual" conditions at the supports. Computer techniques facilitates this type of formulation and also allows to investigate various possibilities of behavior considering the stability of the structure.

The author is planning to further this investigation following these procedures in order to derive a better understanding of the structural action of the masonry dome, and to continue researching on the possibilities offered by the application of the principles of the golden mean to predict the behavior of other masonry structures.

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## *Chapter 3.*

### **The concept of proportion in heritage architecture: a study of form, order and harmony**

This is an ongoing investigation conducted by the author to examine how different concepts of proportion were employed by Western civilization to create “form, order, and harmony” in architectural heritage structures. The study covers the period from Antiquity to the Middle Ages and up to the Renaissance. This is indeed an enormous undertaking in view of the fact that there were so many different systems of proportion employed by such diverse civilizations that makes it impossible, if not futile to attempt to discuss the “history” of proportions in the present article. In view of these practical restrictions, this paper discusses how a particular system of proportion was applied to extant monuments constructed by the ancient civilizations of Egypt and Greece. Future articles will consider Rome, the Middle Ages and the Renaissance and their views towards the concept of proportion in its varied forms.

#### **1. Introduction**

Since ancient times, the practice of architecture and building construction was based primarily on craft tradition and starting from this tradition architectural styles and forms were created by employing systems of proportion. These architectural styles and forms, developed over the centuries with hardly any or no application of analytical structural mechanics. Higher civilizations believed in an order based on numbers and relations of numbers, and they sought and established a harmony, often a mystical one, between universal and cosmic concepts and the life of man. As long as monumental art and architecture were devoted to religious, ritual, cosmological and magical purposes they had to be expressive of this order and harmony by means of proportions. Thus,

architectural forms were created by applying different systems of proportion to the design of historic constructions. A design system was created that established the structural stability, aesthetics of order and harmony, and topology of the monument. This development brought about design concepts and methods of construction employed in architectural heritage. The ratio or proportion commonly known as “The Golden Section” appears to be the principal invariant among the various systems of proportion employed by the builders of heritage architecture.

The present work discusses in a condensed form, how the Golden Section was employed by the Egyptians in the Great Pyramid of Khufu (Cheops) and used by the Greeks in the Parthenon.

## 2. Concept of proportion

The general concept of proportion defines a combination or relation between two or several ratios. The notion of proportion then, follows immediately that of the ratio. A ratio is the quantitative comparison between two aggregates belonging to the same kind or species. If we are dealing with segments ‘A’ and ‘B’ of a straight line, the ratio between these two segments will be symbolized by A/B measured with the same unit. This ratio A/B has all the properties of a fraction. As declared by Euclid in his Elements: “Proportion is the equality of two ratios”.

Systems of proportion [1] can be classified according to a *practical method* based on the repetition of geometric shapes or by the type of *mathematical* relationships employed to obtain the dimensions of the shapes. A brief discussion follows.

### 2.1 Systems of proportion

The systems of proportion which evolved over time can be classified in two general categories, either by the *practical method* which is used to put them into effect, or by the type of *mathematical* relationships which they embody.

According to the *practical method* these systems consists of geometrical systems (based on “shapes”) which aim directly at the repetition of similar shapes; patterns of proportional relationships developed among the dimensions. The *mathematical method* divides proportions into systems using dimensions (based on “mathematical” relationships) which are commensurable and which are related by geometric progressions based on whole numbers (rational numbers); and systems using dimensions which are often incommensurable, and which are related by geometric progressions based on other numbers (irrational numbers).

As indicated earlier, it is not our purpose here to study the history of proportion but to discuss how a particular system of proportion was employed in heritage architecture.

In general, the most important systems of proportions were based on geometric, arithmetic and harmonic relationships.

### **2.1.1 Geometrical proportion**

The name of the geometrical proportion ( $A/B = C/D$ ) was in Greek and in Vitruvius, *analogia*; harmoniously ordered or rhythmically repeated proportions or analogies introduced “Symmetry” and analogical recurrences in all consciously composed plans. As defined by Vitruvius: “Symmetry” resides in the correlation by measurement between the various elements of the plan, and between each of these elements and the whole...as in the human body...it proceeds from proportion - the proportion which the Greeks called *analogia* – it achieves consonance between every part and the whole. This symmetry is regulated by the modulus, the standard of common measure of the work considered, which the Greeks called the Number. When every important part of the building is thus conveniently set in proportion by the right correlation between height and width, between width and depth, and when all these parts have also their place in the total symmetry of the building, we obtain eurhythmy.”

It is the modulus (the common measure) or the Number to the Greeks, that regulates the symmetry of design.

If we have established two ratios  $A/B$  and  $C/D$  between comparable quantities, then the equality  $A/B=C/D$  ( $A$  is to  $B$  as  $C$  is to  $D$ ) is said to be a proportion. The four magnitudes  $A$ ,  $B$ ,  $C$ ,  $D$  are connected by the proportion. This is the geometrical proportion, called discontinuous in the general case when  $A$ ,  $B$ ,  $C$  and  $D$  are different. If two of these numbers are identical it is then called a continuous geometrical proportion, as for example when  $A/B = B/C$ ; or  $B^2 = AC$ . Then  $B = \sqrt{AC}$ , is called the proportional or geometrical mean between  $A$  and  $C$ .

### **2.1.2 Arithmetic proportion and harmonic proportion**

These systems of proportion are based on arithmetic and harmonic relationships. In the arithmetic proportion the middle term overlaps the first term by a quantity equal to that by which it is itself overlapped by the last term, as in 1, 2, 3. In the harmonic proportion the middle term overlaps the first one by a fraction of the latter equal to the fraction of the last term by which the last term overlaps it, or 2, 3, 6 (here the fraction is  $\frac{1}{2}$ ) or 6, 8, 12 (here the fraction is  $\frac{1}{3}$ ).

All of these systems of proportions are a recurring theme throughout the history of architecture, but it is the geometrical proportion, discontinuous or continuous, which is generally used in architecture. In particular, the application of the

Golden Section, to architectural heritage structures in order to create “form, order, and harmony” in the design of these monuments.

### 3. The Golden Section $\Phi$

Starting as early as Egyptian times the design of the most important monuments of every great period or style of architecture were executed according to a very subtle and rational “dynamic symmetry” in which the special properties of the Golden Section were used to obtain the most flexible and varied “eurhythmy” [2].

The Golden Section is a proportion found in nature. Apparently, it was developed by the Ancient Greeks who applied it to designing their temples. It is described as *“the whole is to its larger section, what the larger section is to the smaller”*. It is also known as the Golden Mean, the Golden Ratio, or the Divine Proportion. As shown in Figure 1, it is a way to divide a line in such a way as to create an ideal relationship between the parts.

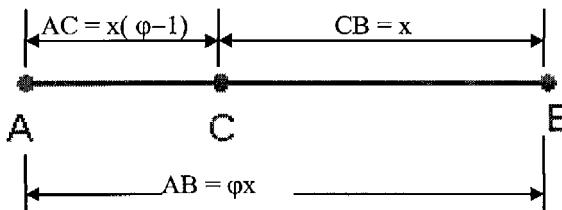


Figure 1. Golden Section of a line

From the figure,  $AB/CB = CB/AC$  or,

$$\frac{\varphi x}{x} = \frac{x}{x(\varphi - 1)}, \text{ that is:}$$

$$\varphi^2 - \varphi - 1 = 0 \quad (\text{solving for } \varphi \text{ we obtain } \varphi = \frac{1 + \sqrt{5}}{2} = 1.61803..) \quad (1)$$

The qualities of the Golden Section  $\varphi$ , (given the name Phi probably after the Greek sculptor, Phidias) were central to the numerological philosophy of Plato and Pythagoras. The mathematician Filius Bonacci (Fibonacci) wrote a treatise on the number series related to the Golden Section.

0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597,.....

If we take the ratio of two successive numbers in Fibonacci's series, we find:

$$1/1=1; \quad 2/1=2; \quad 3/2=1.5; \quad 5/3=1.666..; \quad 8/5=1.6; \quad 13/8=1.625;$$

$$21/13=1.61638..; \quad 34/21=1.61904..;$$

$$144/89=1.61797..; \dots; 610/377=1.61803..; \dots; 1597/987=1.61803\dots$$

The ratio converges to a value very close to the Golden Section ( $\phi$ ) just as for the "line" above!

Throughout history Phi has been observed to evoke emotion or aesthetic feelings within us. The ancient Egyptians used it in the construction of the great pyramids and in the design of hieroglyphs found on tomb walls. The Greeks studied Phi closely through their mathematics and used it in their architecture.

The Parthenon at Athens is a classic example of the use of the Golden Rectangle. The Golden Rectangle, shown in Figure 2, is produced mathematically when the side DC of a square ABCD is bisected at X and an arc with radius XB is swung on to DC at F. The resultant rectangles AEFD and BEFC are both golden rectangles whose sides conform to the golden section, having the ratio 1: 1.618.

That is:  $DF/AD = BC/CF = 1.618$ .

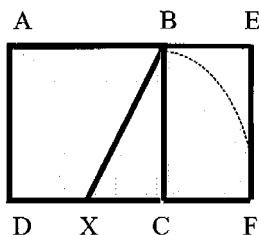


Figure 2. Golden Rectangle

The golden section has been utilized by numerous artists and scientists since (and probably before) the construction of the Great Pyramid of Khufu (c.2600 BC). As scholars and artists of past eras discovered (i.e.: Pythagoras, Plato, Leonardo Da Vinci), the intentional use of the golden section in art of various forms expands our sense of beauty, balance, and harmony to optimal effect.

### 3.1 The Golden Section Phi $\Phi$ in the ancient world

#### 3.1.1 Egypt

Although pyramids may be found in other parts of the world, notably in Central America, it is this remarkable collection of pyramids, particularly the Great Pyramid of Khufu at Giza, Figure 3, which has most captured the world's

imagination. According to historical sources [3], the ancient Greek historian Herodotus (c. 484 – 420 BC) was told by Egyptian temple priests that the Great Pyramid was constructed in such a way that the area of each of its four faces is equal to the square of its height. Following Herodotus argument and referring to Figure 4, let the height of the pyramid be  $h=\sqrt{x}$  units, and the length of the base edges be 2 units. Then the square of the height is  $h^2 = x$ , and the slant height (or apothem) of the pyramid is  $x$  units, as this gives the area of the face to be  $x$ .

$$\text{Thus: } x^2 = 1^2 + [\sqrt{x}]^2 \quad (2)$$

$$\text{or, } x^2 = 1 + x \quad (3)$$

Therefore,

$$x^2 - x - 1 = 0 \text{ (compare with equation 1).} \quad (4)$$



Figure 3. The Great Pyramid of Khufu, Egypt

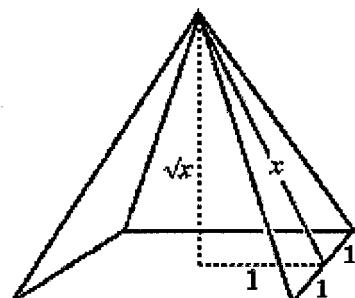


Figure 4. Egyptian Triangle

This equation is the defining relation for 1.618033... the golden section. In other words, for a pyramid of a square base of length 2, the height of the pyramid is  $\sqrt{\phi}$  and the slant height (apothem) is  $\phi$ .

The original dimensions of the pyramid in royal cubits were: the base length 440 (231m), height 280 (147 m), and apothem 356 (186.9m), where 1 royal cubit = 0.525 meters, the ‘standard’ length of a forearm from elbow to finger. These dimensions are a consequence of the proportions 1,  $\sqrt{\phi}$ ,  $\phi$  (respectively: half the side of a square base, the height, and the apothem).

### 3.1.2 Greece

The Parthenon was built during the 5th century B.C. in the ‘Golden Age of Pericles’ when Athens was at the height of its glory. It was a temple dedicated to the honor of Athena Parthenes, the patroness of Athens, whose highly ornate 12-meter high statue was erected in an inner chamber. The building was a proportioned sculpture constructed to house the goddess, rather than a place of worship as one normally thinks a temple to be. The architects Iktinos and Kallikrates worked under the chief designer, the sculptor Phidias.

The Parthenon is an interesting example of a mathematical approach to art. Once its ruined triangular pediment is restored, the ancient temple fits almost precisely into a golden rectangle, Figure 5. Further classic subdivisions of the rectangle align perfectly with major architectural features of the structure.

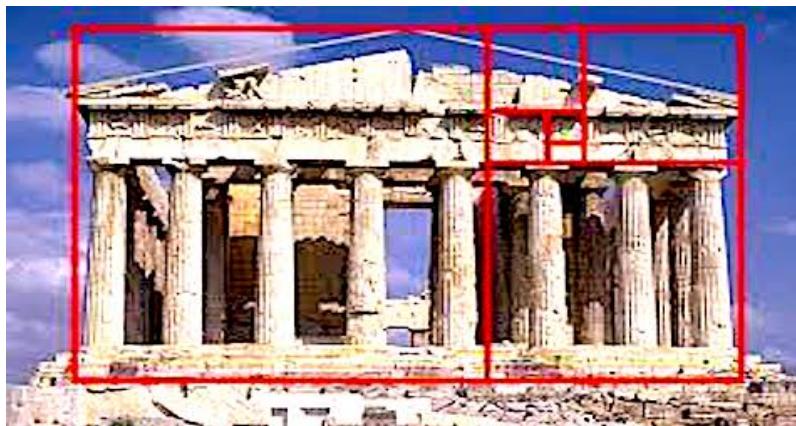


Figure 5. The Parthenon, Athens.

As shown in Figure 6, the facade of the Parthenon apparently was designed around the proportions of two large and four small Golden Section, or  $\sqrt{5}$ , rectangles (refer to Figures 6a, 6b), placed above four squares. These proportions are in agreement with the Golden Rectangle, shown in Figure 2.

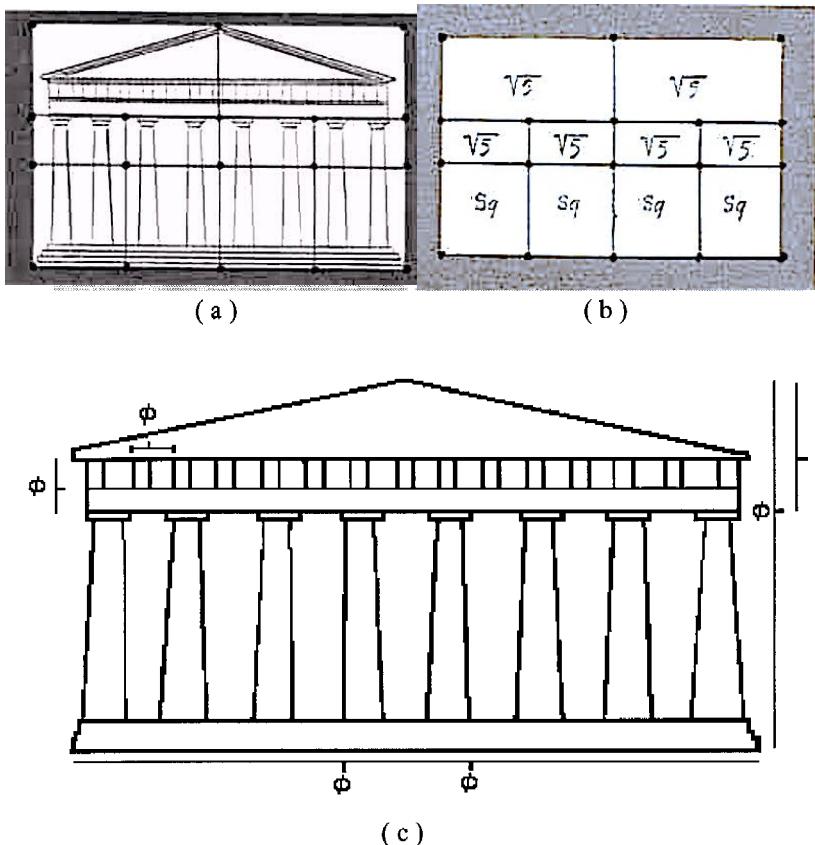


Figure 6. The Parthenon and the Golden section

#### 4. Conclusion

Since the ancient world, different concepts of proportion were employed by Western civilization to create “form, order, and harmony” in architectural heritage structures. The ratio or proportion commonly known as “The Golden Section” appears to be the principal invariant among the various systems of proportion employed by the builders of heritage architecture.

As we have seen, important examples of the use of the Golden Section in ancient architecture are the Great Pyramid of Khufu at Giza, Egypt and the Greek Parthenon in Athens.

The Golden Section is clearly a pervasive topic in heritage architecture. Through its fundamental relationship with the Fibonacci sequence, found throughout nature and art, the Golden Section may very well be a definitive characteristic of architectural design and an element of aesthetic quality.

It is possible that the aesthetic properties associated with the Golden Section were appreciated in ancient times, although its mathematical properties were not necessarily fully understood.

Nonetheless, and all the more beautifully, the mathematical relations in the form of the Golden Section proportion are there in the Great Pyramid of Khufu and in the Parthenon and they are unquestionably true.

Our fascination with the golden section and all it represents, as it have been in the past, will continue to be of interest for mathematical speculation and design considerations in heritage architecture.

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## *Chapter 4.*

# **Evolution of architectural forms of historic buildings**

## **Abstract**

This paper examines the evolution of architectural forms of historic masonry buildings up to the Middle Ages. Consideration is given to those building forms or structures that contributed most to the stability, aesthetics and topology of the monument. It also discusses how this development brought about structural concepts and methods of construction employed in architectural heritage. The thesis proposed in this paper emphasizes the important role played by the development of architectural forms in defining structural aspects of historic buildings.

## **1 Introduction**

The analysis of architectural forms provides an insight into past cultures and eras. Behind each of these forms and styles lies neither a casual trend nor a vogue, but a period of serious and urgent experimentation directed toward answering the needs of a specific way of life. Climate, methods of construction, available materials, and economy of means all impose their dictates. Each of the most significant architectural forms has been aided by the discovery of new construction methods. Once developed, a method survives tenaciously, giving way only when social changes or new building techniques have reduced it.

This process is exemplified by the evolution of architectural forms of historic masonry buildings.

Three important developments, in the form of structural systems, contributed to the determination of building forms: the post-and-lintel, or trabeated, system; the arch (vaulting) system, either the cohesive type, employing plastic materials hardening into a homogeneous mass, or the thrust type, in which the loads are received and counterbalanced at definite points; and the Gothic-skeletal system, in which the basic building components are combined to form an organic structural unity.

In the following discussion, a brief account of the development of architectural form is given considering these basic structural systems to be determinant in the evolution of architectural forms.

## 2 Evolution of architectural forms

### 2.1 Historical notes

Since ancient times, available building materials and tools have determined or modified architectural forms. Methods of construction and structural considerations also contributed to the evolution of architectural form. From the Walls of Jericho (8000 BC) to the Gothic cathedrals of the XIII century, this evolution of building forms or structures can be seen as a continuous development of structural elements whose basic function is to support loads (columns, pillars, walls) and those elements that perform principally a spanning function (beams, arches, vaults and domes).

The practice of architecture and building construction was based on craft tradition dating back to antiquity. The “stacking system” prevailed with the ziggurats of Mesopotamia, Fig. 1, and the pyramids of Egypt, Fig. 2.



Figure 1: Ziggurat, Mesopotamia



Figure 2: Pyramids, Egypt

In Egyptian architecture, the earliest extant structures (erected by 3000 BC), to be called architecture consisted of the post-and-lintel system, Fig. 3. This trabeated system was employed exclusively and produced the earliest stone columnar buildings in history. Later, after generations of experimentation with buildings of limited variety, the Greeks gave to the simple post-and-lintel system the purest, perfect expression it was to attain as in the Parthenon, Fig. 6.

For a long time, the classical Greek system of the “orders” became the most visible contents of architectural form emphasizing composition and the concepts of proportion and harmony, thus representing the formal characteristics of architectural tradition. These orders: Doric, Ionic and Corinthian, were purely ornamental in nature and did not perform a structural function. The tradition of Greek architecture was preserved but the recognition that the orders were nothing but ornament encouraged architects to take liberties in their designs. Architraves could be interrupted, curved or bent upwards into arches.



Figure 3: Post and Lintel, Egypt

Arches have been built since prehistoric times. The Egyptians, Babylonians, and Greeks used the arch and the corbel, generally for secular structures, such as storerooms and sewers. Roman architecture, borrowing from the Etruscans and combining the columns of Greece and the arches of Asia, produced a wide variety of monumental buildings throughout the Western world. Their momentous invention of concrete enabled the imperial builders to exploit successfully the vault construction of Asia and to cover vast floor spaces with great vaults and domes, as in the Pantheon.

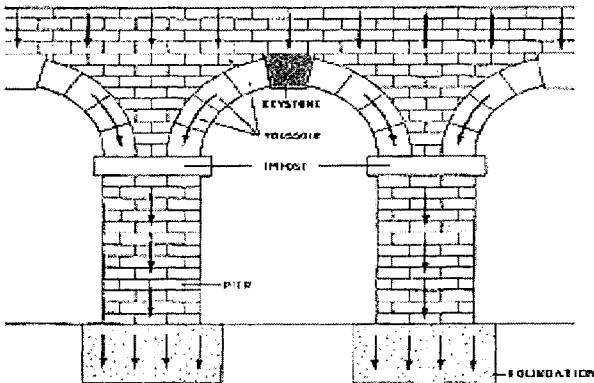


Figure 4: Roman Arch

The Romans were the first to develop the arch, Fig. 4, on a massive scale. They realized the structural potential of the arch and its derivatives, the vault and the dome, by incorporating these elements into their constructions.

Byzantine architects experimented with new vaulting principles and developed the pendentive, Fig. 5, used brilliantly in the VI century for the Church of Hagia Sophia in Constantinopla.

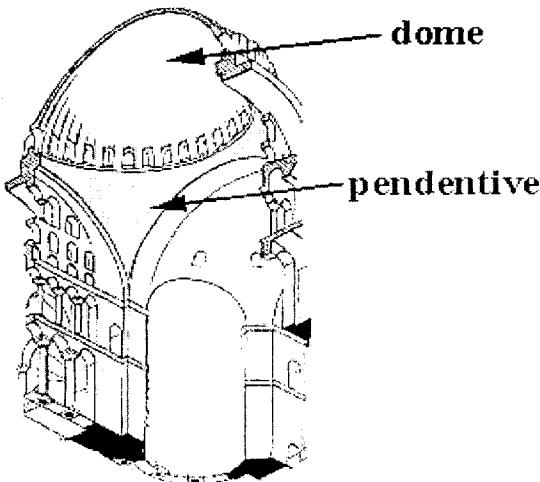


Figure 5: Dome, Pendentive

The Romanesque architecture of the early Middle Ages was notable for strong, simple, massive forms and vaults executed in cut stone. In Lombard Romanesque (XI cent.) the Byzantine concentration of vault thrusts was improved by the device of ribs and of piers to support them. The idea of an organic supporting and buttressing skeleton of masonry, here appearing in embryo, became the vitalizing aim of the medieval builders. In Gothic architecture it emerged in perfected form, as in the nave of Notre Dame, Fig. 7.

The birth of Renaissance architecture (XV cent.) inaugurated a period during which the multiple and complex buildings of the modern world began to emerge, while at the same time no new and compelling structural conceptions appeared.

Nevertheless, the Renaissance brought about a new interest in the accomplishments of antiquity, especially in Italy. Ancient works of art and buildings became objects of study, prompting a search for writings on architectural theory and building construction. Structural mechanics began to develop largely because of the increasing demand for a rational analysis connected with the building of masonry stone vaults. The main motivation for the early development of structural theory was to understand and control the fracture of materials as evidenced by Leonardo DaVinci's studies and later by Galilee's experiments. These developments undoubtedly contributed to the creation of modern structural theory.

### **2. 1.1 Stone architecture**

The period running from the Paleolithic era to the Middle Ages is characterized by the use of masonry, and in particular, stone in architecture, heralding future uses of the material. For example the horizontal stone in dolmens (chamber tombs walled and roofed with megaliths) would later form the basis for the lintel principle used so magnificently by the Egyptians and the Greeks.

Following the era of megalithic monuments, there was a move towards a more elaborate form of architecture with a religious function. As previously mentioned, stone architecture began with a "stacking" system, of which the Egyptian pyramids are typical examples. The Romans brought about developments in stone construction through a rational use of the arch principle and through the construction of domical vaults made of stone.

Romanesque architecture in the Middle Ages went on to draw from Roman experience and made few changes to the basic principles. The feat of conquering verticality, intended as an expression of religious fervor, led cathedral builders of the Gothic period to develop the pointed arch and to discover the flying buttresses system that they made great use of from an aesthetic as well as a structural point of view.

Vaults were used to span greater lengths of space along the naves and aisles of the cathedrals. With these efforts to “lighten” structures, masonry stone-construction reached the limits of its potential during the Gothic period. The Sainte-Chapelle, built in Paris in 1246, represents the height of achievement in this respect.

## 2.2 Structural aspects

### 2.2.1 Function and structural characteristics

Function refers to the specific purpose for which an element is used in a structure. That is, some elements are employed to carry loads while other elements are used principally to span a certain space. Structural characteristics are those referring to strength, stiffness, and stability. The structure must be strong enough to carry loads; it must not deflect unduly; and it must not develop unstable displacements. Strength and stiffness are not conditions of great relevance in the design of masonry, whereas those pertaining to stability are of primary importance, particularly when designing curved-shaped masonry structures. These pictures show how builders of different periods overcame the difficulties of spanning a large ground area, and in the end achieved a logical and efficient construction.



Figure 6: The Parthenon



Figure 7: Notre Dame

The post-and-lintel system of a Greek temple and hence its shape, Fig. 6, were determined by the strength of its material, for the space between any two columns could be no greater than the longest possible (strong enough) stone crossbeam. Thus, the post-and-lintel type of construction limited further development in the topology of the architectural form.

The Gothic builder, on the other hand, solved the spanning problem in a different way by relying on the use of the vault, as illustrated in Fig. 7, by the Nave of Notre Dame, Paris. The master builder also depended on strength but, much

more so, on the stability of the masonry structure for the shapes and sizes of the building components of the structure.

Masonry vaults constructed of numerous blocks of material pressing against one another exert not only the accumulated downward weight of the material and of any superimposed load but also a lateral thrust, Fig. 8, or tendency to spread outward, Fig. 8b. To avoid collapse due to instability, adequate resistance against this thrust must thus be concentrated at the haunches of the vault.

The resistance may take the form of thickened walls at the haunches, Fig. 8c; or of buttresses placed at points of concentrated thrust as in Romanesque, Fig. 8d, and Gothic architecture using flying buttresses, Fig. 8e. Metal ties, Fig. 8f, can also be used to resist the lateral thrust.

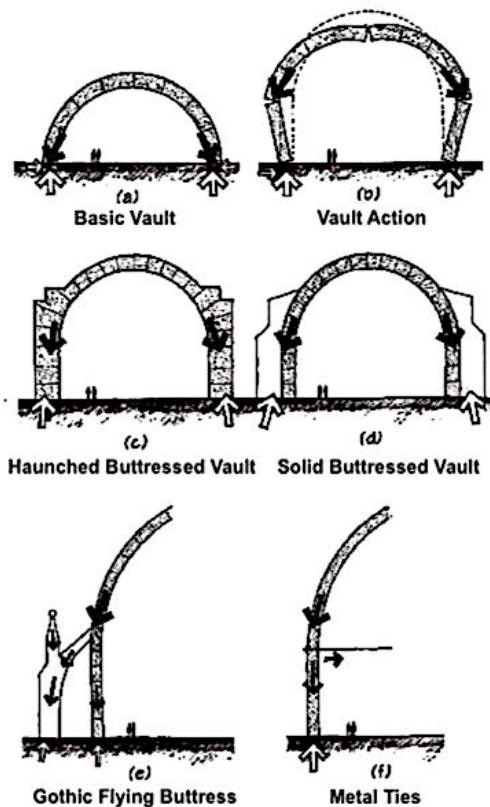


Figure 8 : Thrust in Masonry Vaults

This necessity to resist lateral thrust, mainly from the point of view of stability and not of strength, has controlled the evolution of masonry vaulting (composed of separate units of material), and its use in historic buildings. Consequently, the of masonry vaulting systems that rely on the geometry of its configuration in order to maintain its stability and hence, its structural reliability.

### 2.2.2 The masonry vault

Roman vaults were the basis on which more complex and varied architectural forms were developed during the Middle Ages. Thus, to better understand the evolution of masonry vaulting and how its effective utilization came about, we must start with the Roman barrel vault, Fig.9, in which large blocks of stone, cut to the right size and shape, supported each other to form a tunnel-like structure.

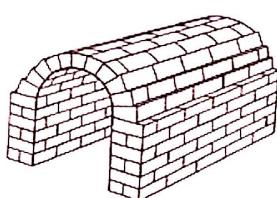


Figure 9: Barrel Vault

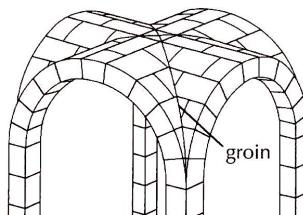


Figure 10: Cross Vault

From the barrel vault, the cross (groined) vault, Fig. 10, is produced by the intersection at right angles of two barrel vaults, creating a surface that has arched openings at its four sides and concentration of load at the four corner points of the square or rectangle. It was found that these cross vaults were much more useful than the barrel vaults, for when risen on pillars to form bays, they could be repeated in order to cover a large area. This allowed for a more spacious and functional structure as was used so effectively in the great Romanesque churches.

Ribs to strengthen the groins and sides of a cross vault were introduced into Romanesque vaulting in the XI century. Further developments created a structural system of using ribs to form a complete organic supporting skeleton thus becoming one of the basic principles of perfected Gothic architecture.

The pointed arch, which was dominant in medieval architecture from the XIII century onward, helped to overcome the difficulties of vaulting oblong compartments exclusively with semicircular sections, as in Roman and Romanesque architecture, and to bring the various ribs of unequal spans to a crown at the same height, Fig. 11. The use of ribs led to increasing complexity in the development of vault forms. The ribs were carried down to the floor as shafts attached to the walls and piers transforming these walls and piers into a glass filled exposed skeleton of arches, columns, and flying buttresses.

The result was an entirely new structural system in which structure, construction and visually expressive architectural forms were integrated into a new aesthetic, thus producing the Gothic-skeletal system of construction. Refer, for example, to Fig. 7, Fig. 8e, and Fig.11 to appreciate this development.

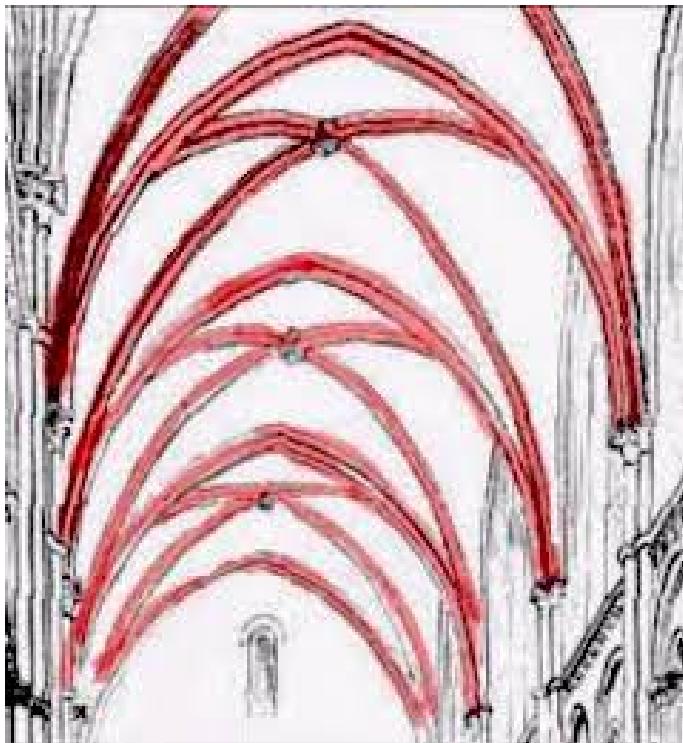


Figure 11: Ribbed Vault

### 3 Conclusion

The evolution of architectural forms has been dependent upon the development of building components whose basic function is to support loads (post-and-lintel system) and those elements that perform a spanning function (vaulting system).

The post-and-lintel system is limited by the strength of the crossbeam spanning the space between any two columns, and being linear in nature, also restrict additional geometric developments in architectural forms.

Vaulting systems, on the other hand, could be erected over vast spaces creating a diverse dimensionality of space allowing for more complex and varied architectural forms, thus producing impressive and monumental structures. It is

this development of curved-shaped structures that is in effect determinant in the continuous evolution of architectural forms.

The development of curved-shaped structural masonry, progressing through the evolution of the vault, brought about significant changes in architectural form and in the structural system of historic buildings. From the invention of the barrel vault to the gradual evolution of (groined) cross -vaulted structures and then, to the creation of the more elaborate ribbed vaulted structures of the Gothic period; vaulting developed in such a mature way so as to control and thus define the architectural forms and structural patterns of construction.

Ribbed vaulting delineated and controlled the Gothic architectural and structural skeletal system of the entire building. All of this structural scheme; consisting of pointed arches, ribs and shafts, piers and buttresses; was designed to support and maintain the stability of the vault. It is then the vaulting system of construction that ultimately produces, and determines the architectural form and structural characteristics of most great historic buildings. This is best exemplified by the Gothic- skeletal system because it is in Gothic that the stone material, function of the elements and the structural behavior of masonry are encountered in their most critical form.

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# About the Author

**Ramón Felipe Borges** is an artist and engineer-scientist who explores different forms of creativity with originality and innovation following his intuition, feelings and the interpretation of experiences in life.

He learned to appreciate that creativity in art, as well as in science, is as much about effect as it is about cognition.

Though an artist by vocation, he became an engineer-scientist by profession. He obtained a Bachelor's Degree in Engineering from the University of Puerto Rico, a Master's Degree from Cornell University, with studies in Architecture; and subsequently, a Ph.D. from Penn State University, where he taught engineering and conducted research for two decades until retiring in 2005. His research was devoted primarily to computer aided design in engineering and heritage architecture of historic buildings.

However, he never lost perspective of his vocation for art and interest in architecture and engineering science and continues to create in these areas of knowledge.

## **CONTRIBUTIONS TO ART:**

By employing the language of art and poetry in a series of books and paintings, Borges examines the mythology and allegories defining our identity, character, and experiences in life. In his works, we take a fascinating journey through poems and pictorial interpretation of myths, metaphors, and memories giving meaning to our history and culture.

### **Books:**

Borges' Paintings  
Myths, Abstracts, Landscapes, and Still Lifes  
Description and Poems  
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Descripción y Poemas  
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mitos, metáforas y recuerdos  
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Saatchi Art, website: <https://www.saatchiart.com/RamonBorges>

### **CONTRIBUTIONS TO ARCHITECTURE AND ENGINEERING EDUCATION:**

Most of his research concerns the area of architectural heritage, engineering education and the history and interpretation of the structural and architectural aspects of culturally meaningful buildings.

A novel approach to his investigation resides in examining how the proportion of the "golden ratio" (The Phi Code) appears systematically coded as a physical phenomenon controlling the growth patterns in nature and determining the behavior of structural elements.

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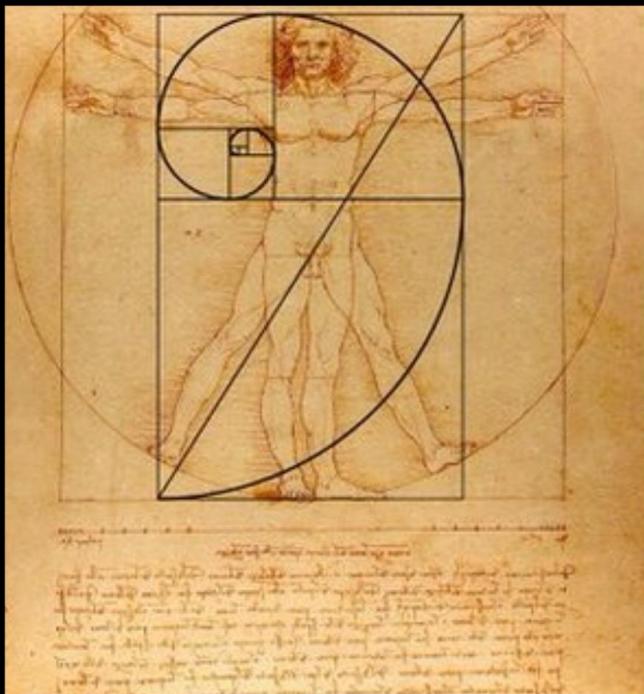
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# THE PHI CODE

Nature, Architecture, Engineering



*“Of all things the measure is man”*  
Protagoras ,Greek philosopher, (490-420 BCE ca.)

RAMÓN FELIPE BORGES, Ph.D

# THE PHI CODE



This book contains papers written and presented by the author at international conferences on Heritage Architecture.

In this volume, the author analyses topics related to structural, historical, and the architecture of culturally meaningful buildings.

The thesis proposed in the book emphasizes the important role played by the "phi-code" in the development of architectural forms and in defining structural aspects of historic buildings.

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